Frequency extention by nonlinear optics

Second harmonic generation

Dielectric susceptibility

Polarization: $\mathbf{P} = \chi(\mathrm{E})\, \mathbf{E}$

$$\mathbf{P} = \chi^{(1)}\, \mathbf{E} + \chi^{(2)}\, \mathbf{E}^2 + \chi^{(3)}\, \mathbf{E}^3 + \ldots$$

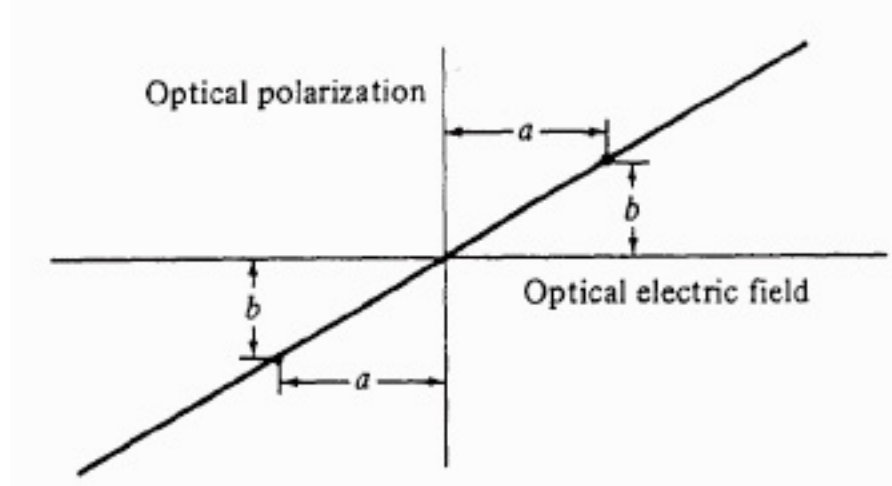
$$\mathbf{P} = \chi^{(1)}\, \mathbf{E} + \chi^{(2)}\, \mathbf{E}^2$$

frequency doubling,
sum and difference frequency mixing,
optical parametric oscillation.

SHG

Linear dielectric Crystall:

$$p(t) = \frac{Ne^2}{m\omega_0^2} E(t)$$



(a)

Nonlinear crystall

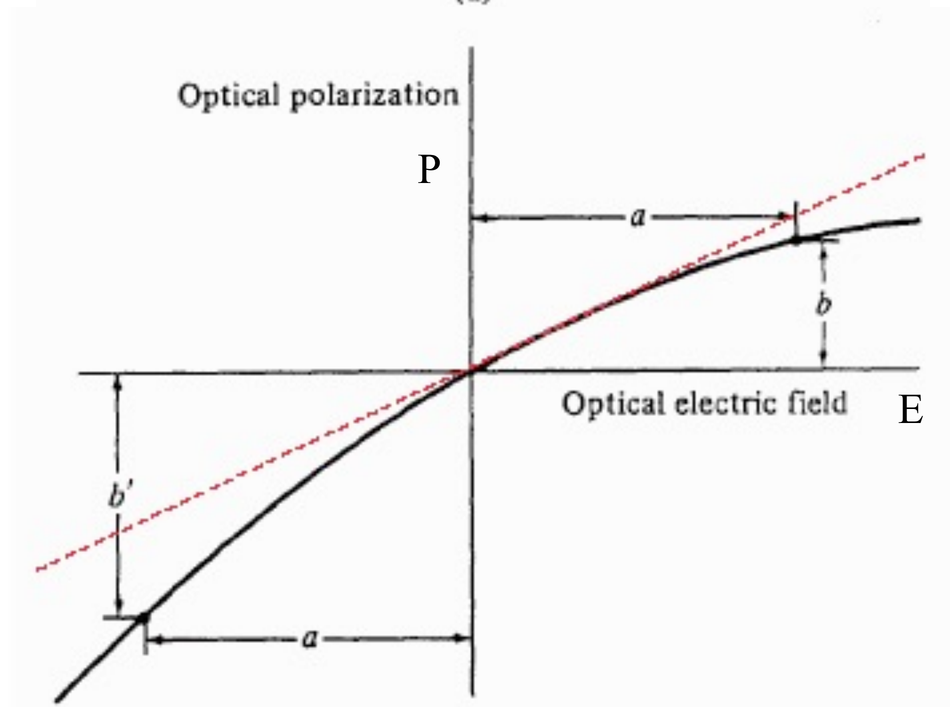
$$E(t) = -\frac{1}{e}(m\omega_0^2 x + mDx^2);$$

$$x(E) \simeq -\alpha E + \beta E^2, \quad \alpha, \beta > 0;$$

$$p(E) = -N \cdot e \cdot x(E) \simeq \chi_1 E - \chi_2 E^2;$$

$$\chi_1, \chi_2 > 0,$$

$$\underline{|p(E)| < |p(-E)|}.$$

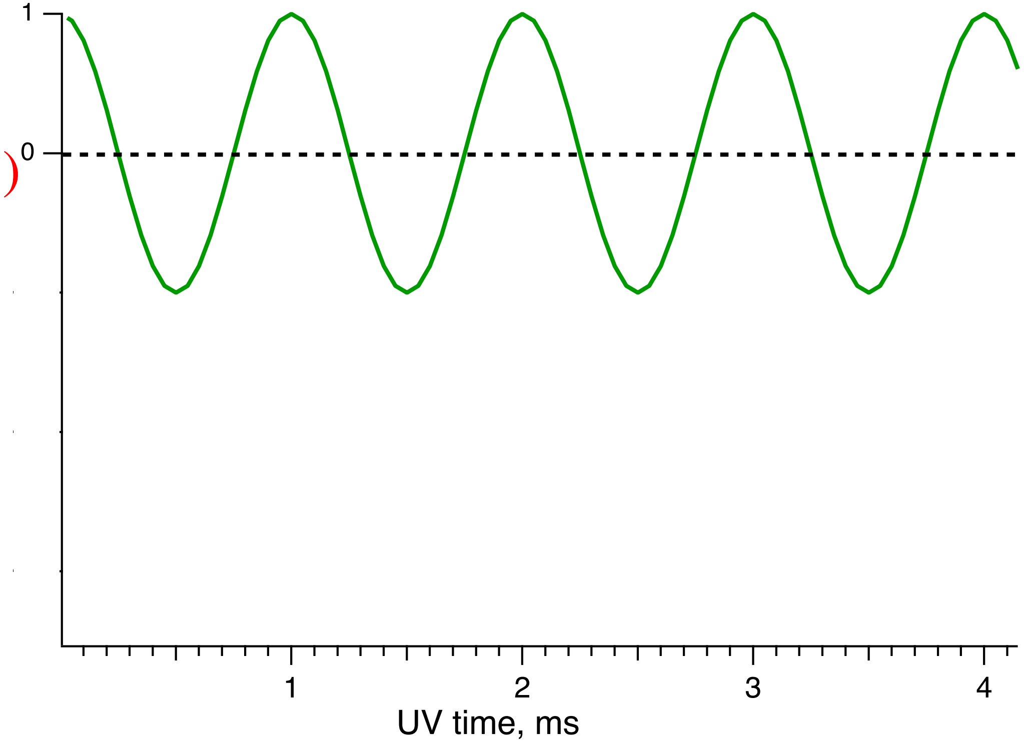


(b)

Optical wave:

$$E = E_0 \cos(\omega t + \varphi)$$

$$P(t) = \chi_1 E_0 \cos(\omega t) - \chi_2 E_0^2 \cos^2(\omega t)$$



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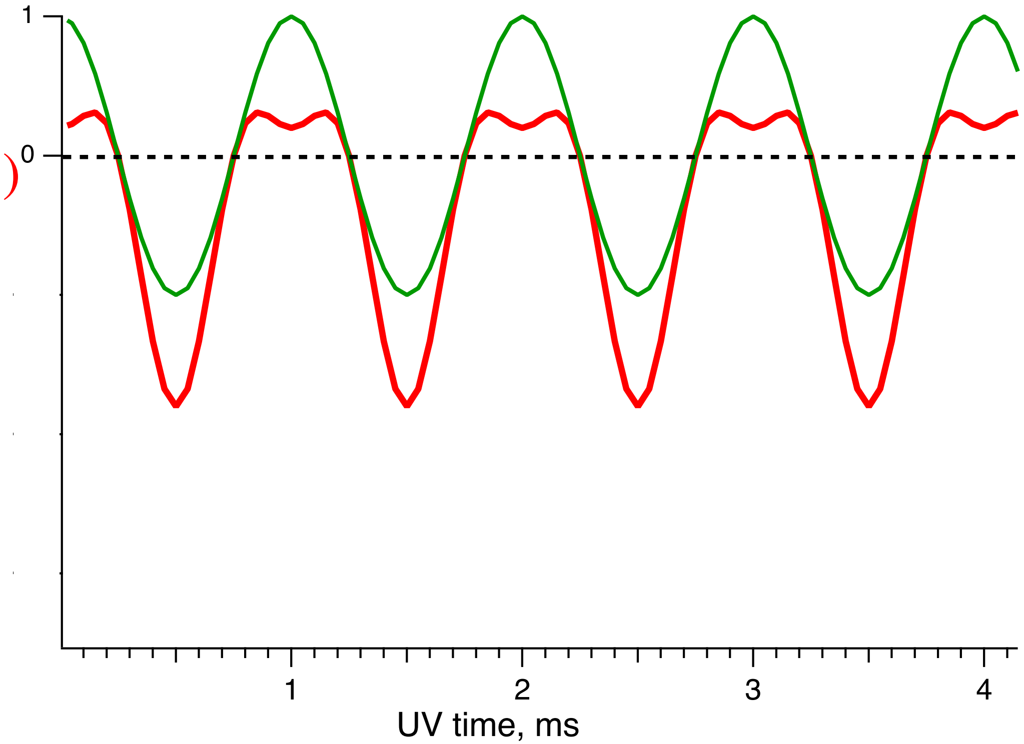

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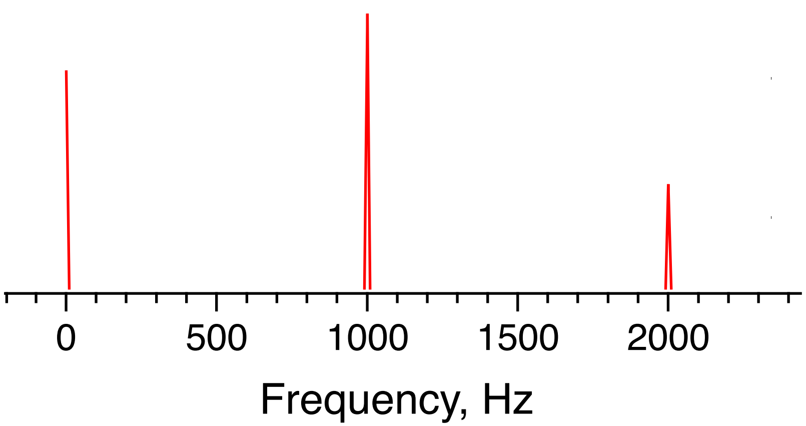
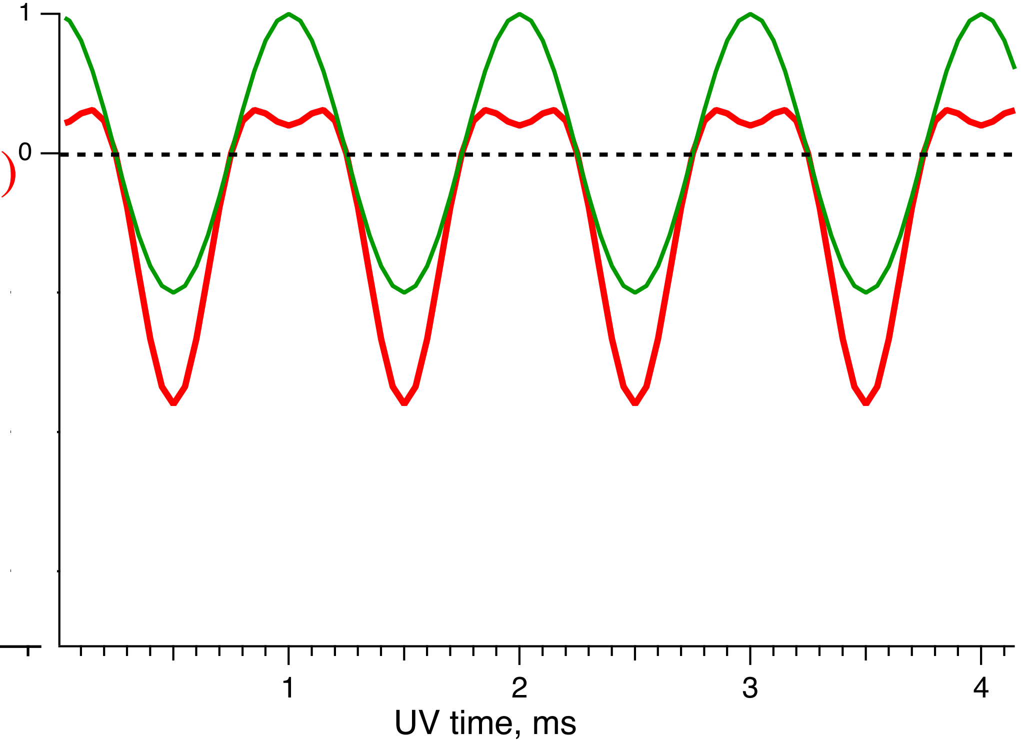
Fourier transformation:





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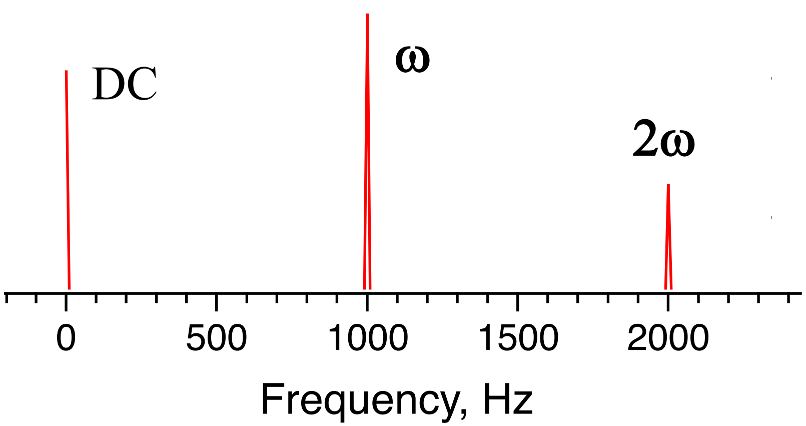
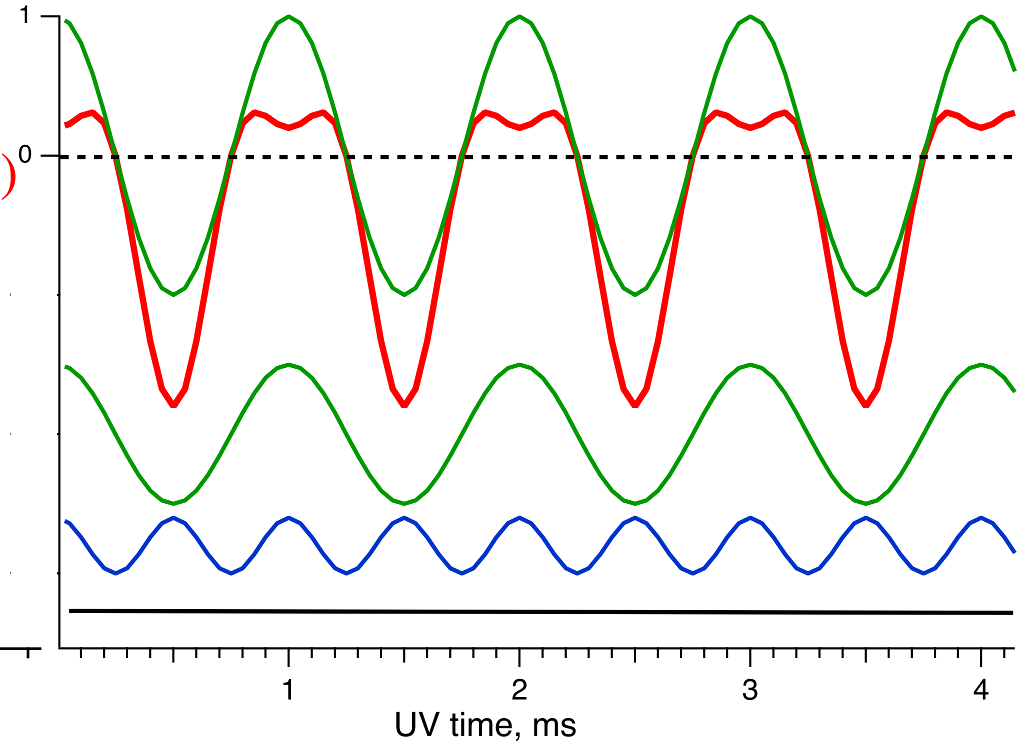




Components of polarization:

- DC component
- Optical frequency
- **Doubled optical frequency** (2nd harmonic)

Difference and Sum frequency mixing

$$P(t) = \chi_1 E_0 \cos(\omega t) - \chi_2 E_0^2 \cos^2(\omega t) + \chi_3 E_0^3 \cos^3(\omega t)$$

Pumping a nonlinear crystal by ω_3 produces **any** frequencies ω_1 and ω_2, which are:

- within the transparency window of the crystal
- $\omega_3 = \omega_1 + \omega_2$ ***DFG***

This process is reversible: $\omega_1 + \omega_1 = \omega_3$ ***SFG***

Efficiency???

Phase Matching in Uniaxial Crystals

SHG efficiency:

$$\eta_{SHG} = \frac{P_{2\omega}}{P_\omega} = 2\left(\frac{\mu}{\varepsilon_0}\right)^{\frac{3}{2}} \frac{\omega^2 d^2 L^2}{n^3} \frac{\sin^2(\Delta kL/2)}{(\Delta kL/2)^2} \frac{P_\omega}{\underline{A}}$$

length

power

area

$$\Delta k = k^{2\omega} - 2k^{\omega};$$

Phase matching: $\Delta k = 0$ <u>That is the two waves move together!</u>

Phase matching length:

$$L_c = \frac{2\pi}{\Delta k} = \frac{2\pi}{k^{(2\omega)} - k^{(\omega)}}, \qquad k^{(\omega)} = \frac{1}{\lambda_n} = \frac{\omega n^{(\omega)}}{c}$$

$$\Delta k = k^{(2\omega)} - 2k^{(\omega)} = (2\omega/c)\,[n^{(2\omega)} - n^{(\omega)}]$$

$$L_c = \frac{2\pi c}{2\omega(n^{(2\omega)} - n^{(\omega)})} = \frac{\lambda}{2(n^{(2\omega)} - n^{(\omega)})}$$

$$L_c \cong 100\mu m$$ Too short!

How to make n(ω) = n(2ω) ?

Birefringent crystals

Birefringent materials

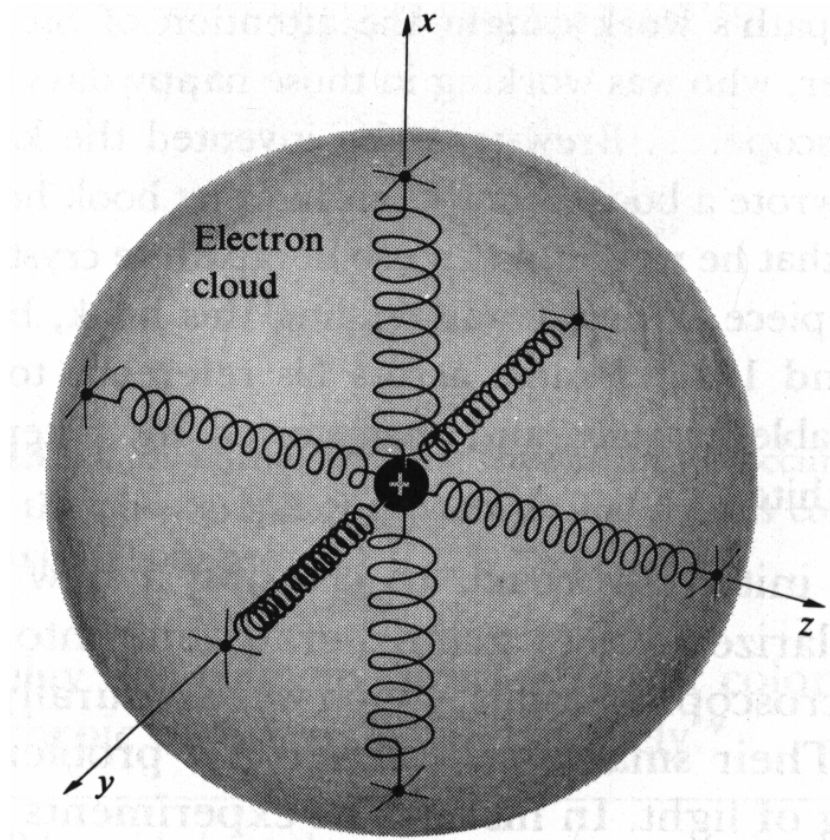
$$n^2(\omega) = 1 + \frac{q_e^{\ 2} N}{\varepsilon_o m_e}\left(\frac{1}{\omega_o^2 - \omega^2}\right)$$

Biaxial crystal

$$n_z \neq n_x \neq n_y$$

Uniaxial crystal

$$n_z \neq n_x = n_y$$

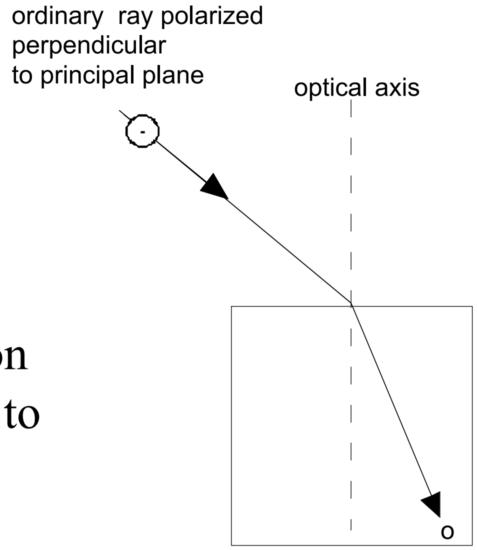


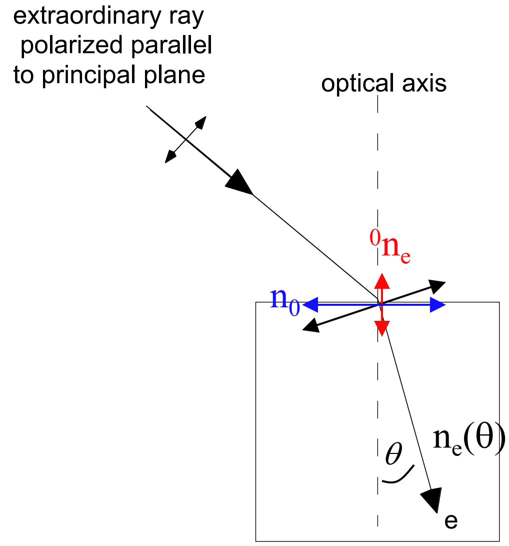
Ordinary index $n_0 = n_x = n_y$;

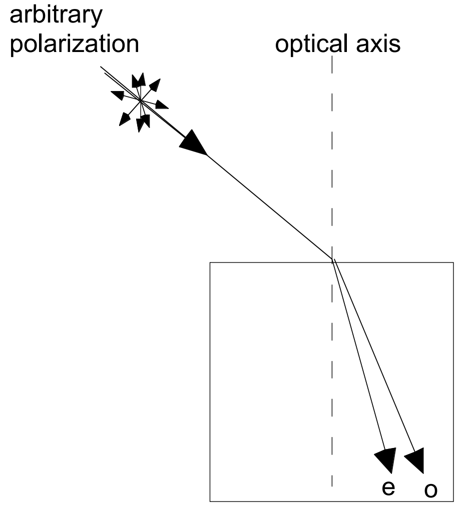
Extraordinary index $n_e = n_z$

Birefringence in uniaxial crystals:

UCs have different refraction index along and orthogonal to the optical axis





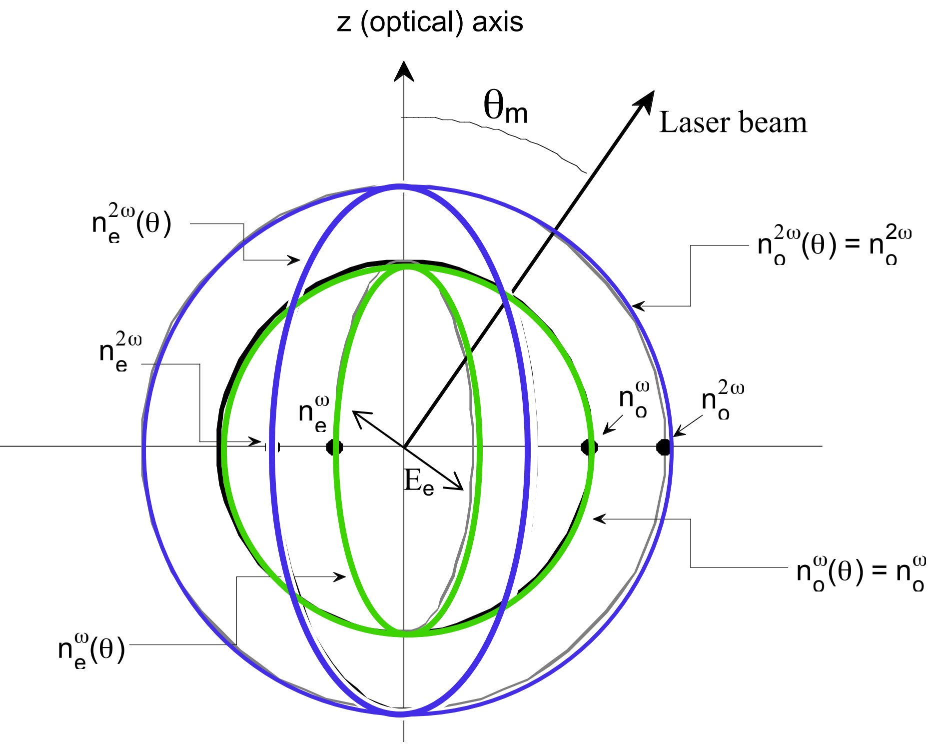


$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_0^2} + \frac{\sin^2\theta}{n_e^2}$$

Birefrigence in Uniaxial Crystals

$\omega = 2\omega$ $\quad k^{(2\omega)} = 2k^{(\omega)}$

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_0^2} + \frac{\sin^2\theta}{n_e^2}$$



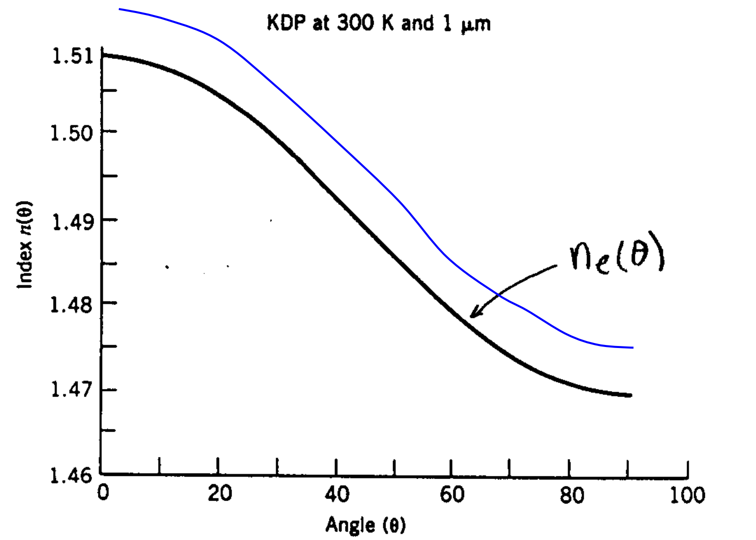


FIGURE 15-8. The variation in the value of the extraordinary index of refraction with propagation angle in KDP at room temperature. The index varies from n_0 at $\theta = 0°$ to n_e at $\theta = 90°$.

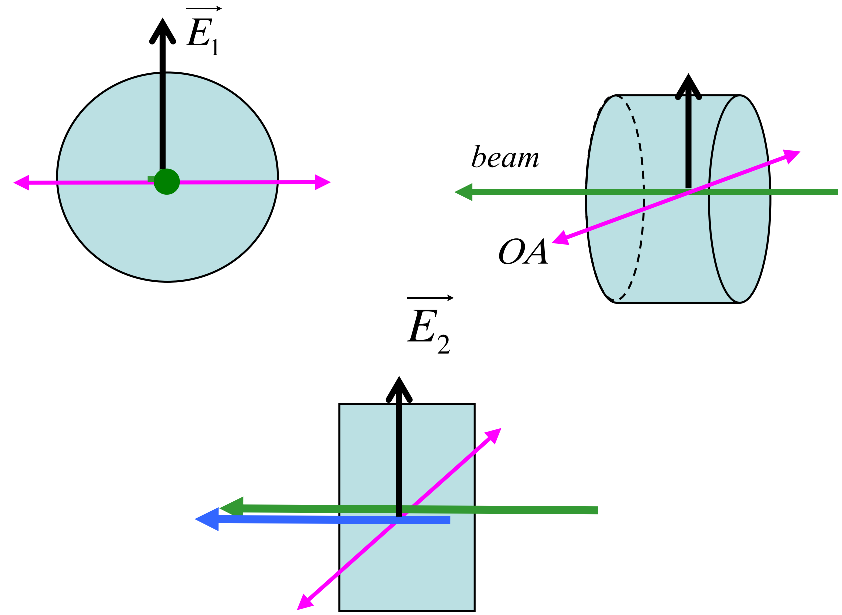
Phase matching: $\quad \Delta k = 0$

$$\eta_{SHG} = \frac{P_{2\omega}}{P_\omega} = 2\left(\frac{\mu}{\varepsilon_0}\right)^{\frac{3}{2}} \frac{\omega^2 d^2 L^2}{n^3} \frac{\sin^2(\Delta kL/2)}{(\Delta kL/2)^2} \frac{P_\omega}{A}$$

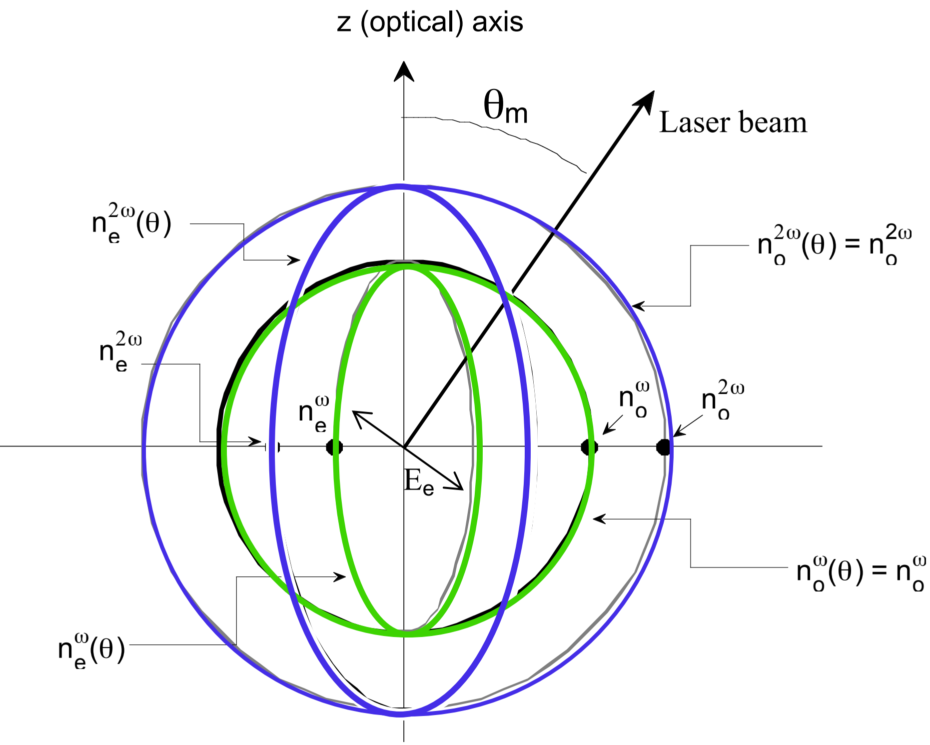
Birefrigence in Uniaxial Crystals

$\omega = 2\omega \qquad k^{(2\omega)} = 2k^{(\omega)}$

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_0^2} + \frac{\sin^2\theta}{n_e^2}$$



Principal plane



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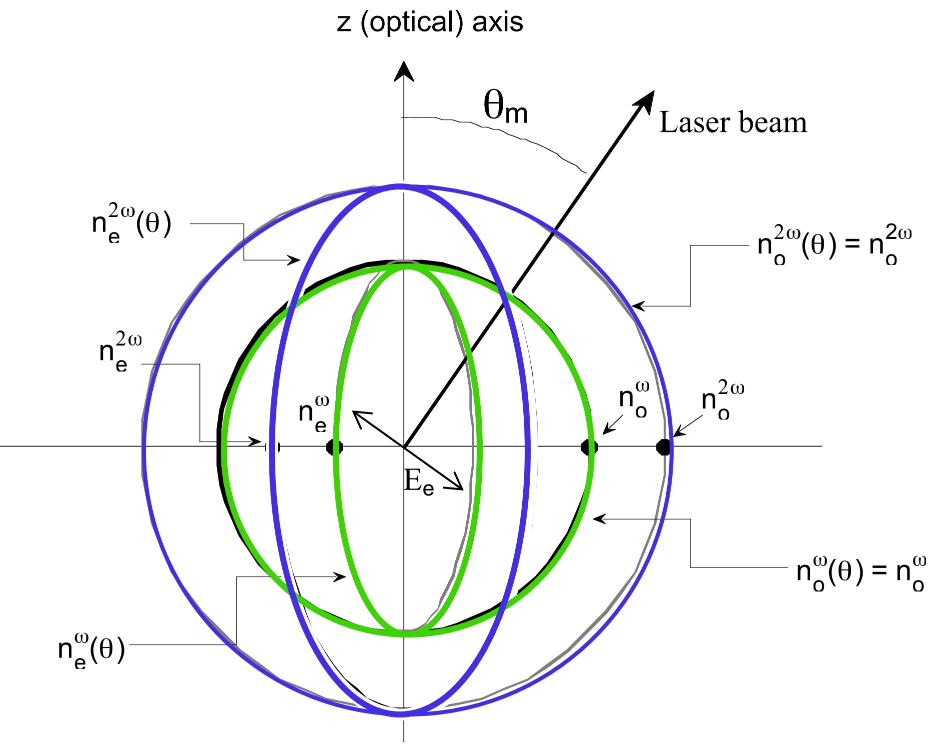
Second Harmonic Generation

$\omega \rightarrow 2\omega$ $\qquad$ $k^{(2\omega)} = 2k^{(\omega)}$

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_0^2} + \frac{\sin^2\theta}{n_e^2}$$



Notations

Fundam, Fundam, SH:

(532 +532 =266nm)

Type I: (rotation around H axis)

ooe= HHV; eeo=VVH

Type II: (rotation around H axis)

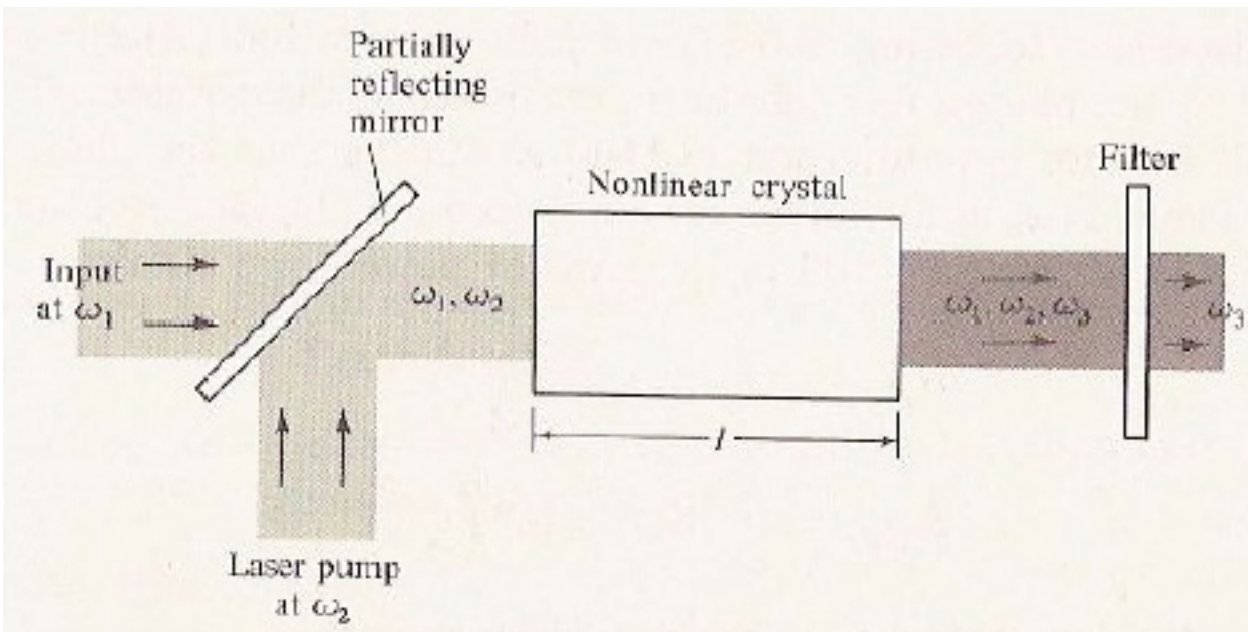
oeo= 45° ,H; eoe=45° V

Frequency Mixing

$$\omega_3 = \omega_2 \pm \omega_1; \qquad \overrightarrow{k_3} = \overrightarrow{k_2} \pm \overrightarrow{k_1}; \quad k(\omega) = \frac{\omega \cdot n(\omega)}{c},$$

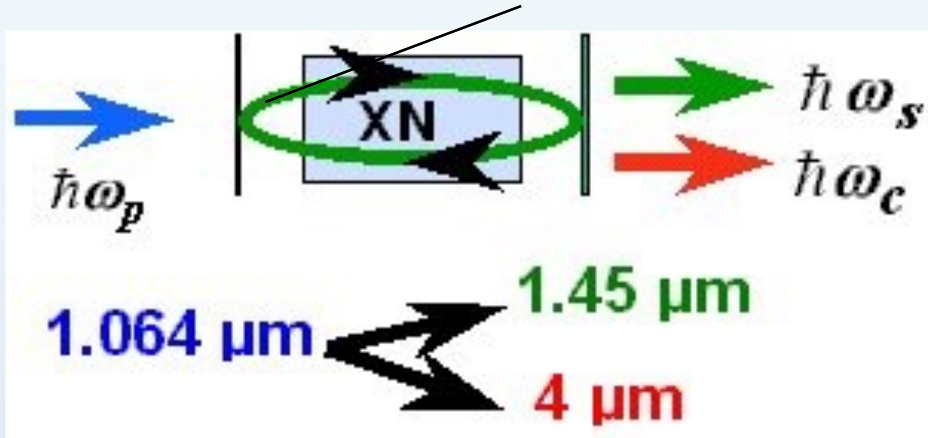
Phase matching conditions: $\quad n_3 \cdot \omega_3 = n_2 \cdot \omega_2 \pm n_1 \cdot \omega_1$

"+" -frequency up conversion, "-" - difference frequency mixing



Optical parametric amplification of ω_1 and ω_3: $P_2 >> P_1$.

Optical Parametric Oscillator (OPO)



The split point is dictated by the angular orientation of the crystal

$$n_p \cdot \omega_p = n_s \cdot \omega_s \pm n_i \cdot \omega_i$$

Mode structure